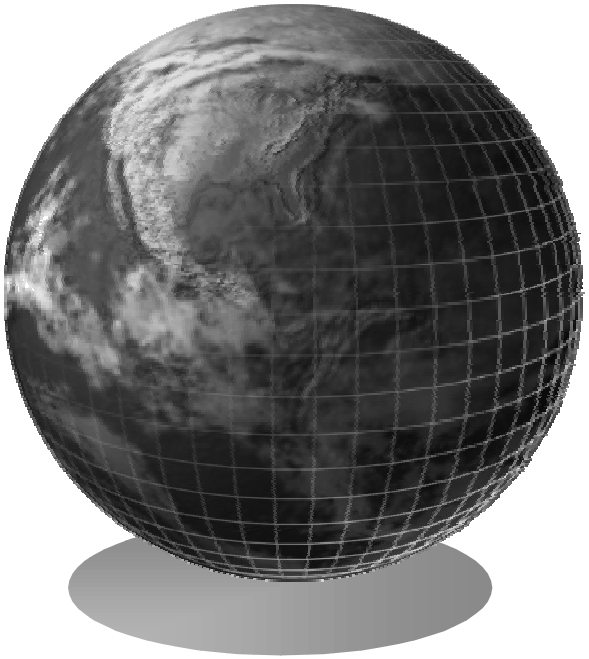
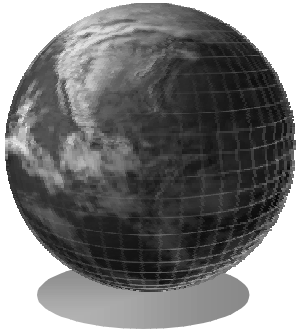
The Small World Problem



Duncan Watts
Columbia University

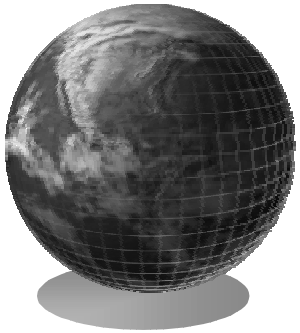
What is The Small World Problem?

- Often referred to as “Six degrees of Separation”
  - “Six degrees of separation between us and everyone else on this planet”
    - John Guare, 1990
- An urban myth? (“Six handshakes to the President”)
- First mentioned in 1920’s by Karinthy
- 30 years later, became a research problem



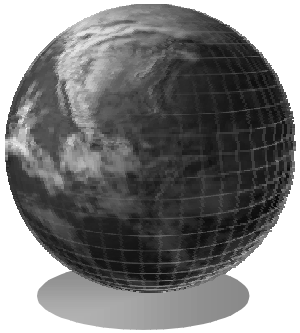
The Small World Problem

- In the 1950's, Pool and Kochen asked "what is the probability that two strangers will have a mutual friend?"
  - i.e. the "small world" of cocktail parties
- Then asked a harder question: "What about when there is no mutual friend--how long would the chain of intermediaries be?"
- Too hard…



The Small World Experiment

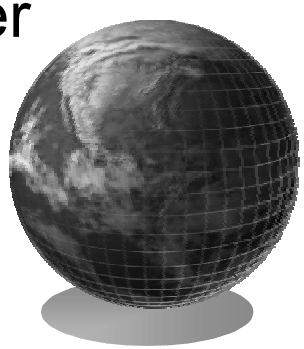
- Stanley Milgram (and student Jeffrey Travers) designed an experiment based on Pool and Kochen's work
  - A single "target" in Boston
  - 300 initial "senders" in Boston and Omaha
  - Each sender asked to forward a packet to a friend who was "closer" to the target
  - The friends got the same instructions



“Six Degrees of Separation”

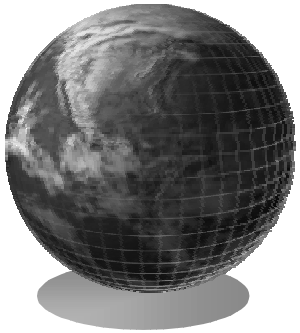
- Travers and Milgram’s protocol generated 300 “letter chains” of which 64 reached the target.
- Found that typical chain length was 6
- Led to the famous phrase (Guare)

- Then not much happened for another 30 years.
  - Theory was too hard to do with pencil and paper
  - Data was too hard to collect manually



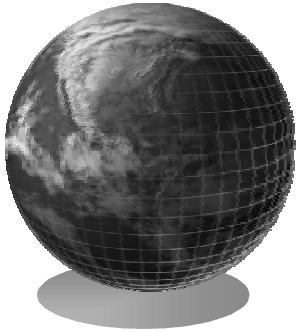
The “New” Science of Networks

- Mid 90’s, Steve Strogatz and I working on another problem altogether
- Decided to think about this urban myth
- We had three advantages
  - We didn’t know anything
  - We had MUCH faster computers
  - Our background in physics and mathematics caused us to think about the problem somewhat differently



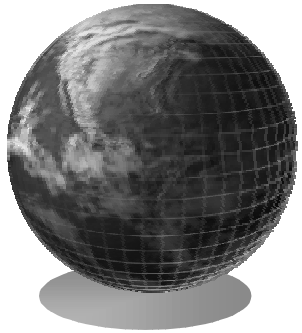
Small World Networks

- We managed to show that if a network has
  - Some source of “order”
  - The tiniest amount of randomness
- It will be a “small-world” network of the kind that Pool and Kochen were looking for
- We also made the prediction that small World Networks should be *everywhere*.

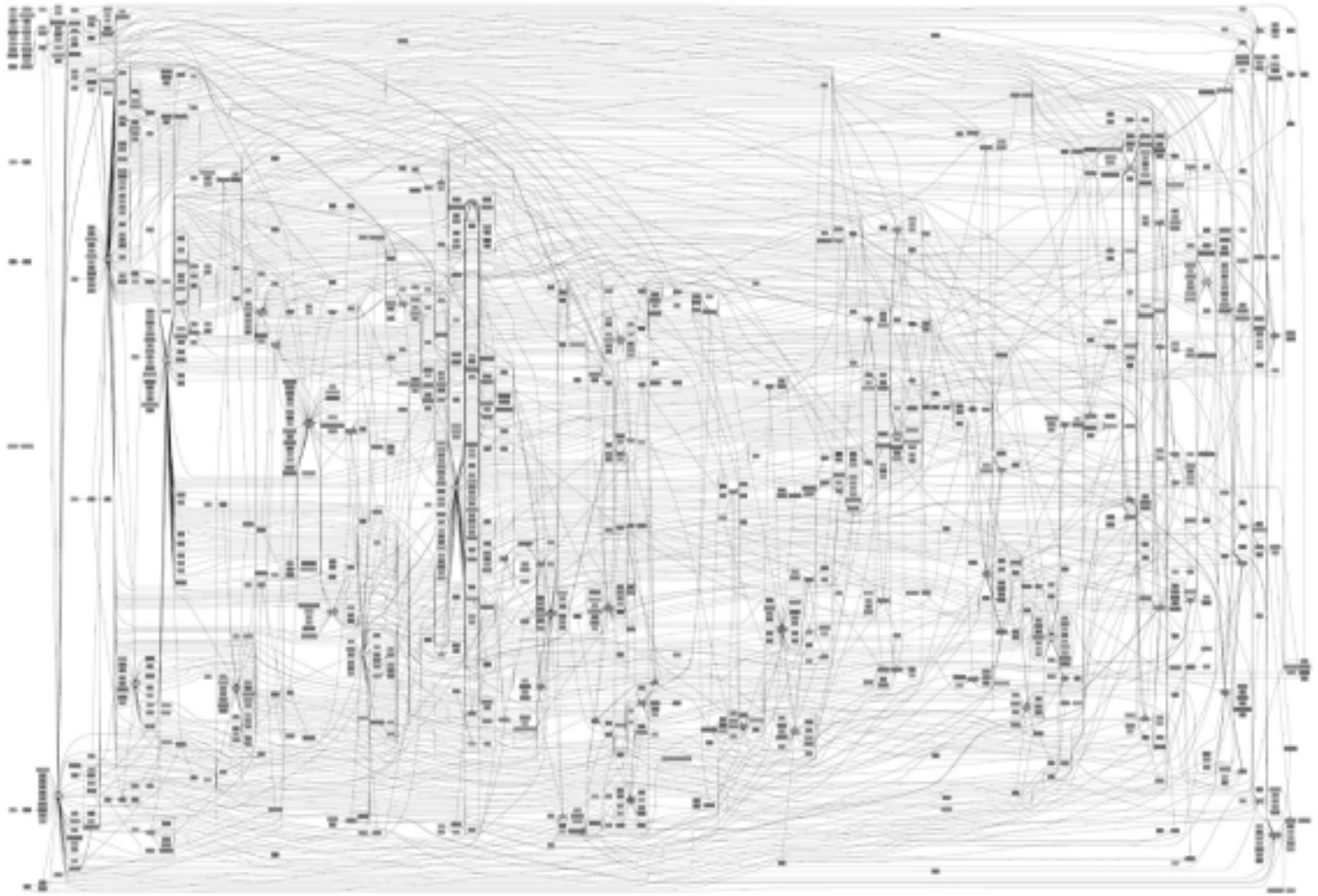


Small-World Networks

- Online social networks
- Email networks
- Networks of movie stars, boards of directors, and scientists
- Power transmission grid of the Western US
- Neural networks
- Genetic regulatory networks, protein interaction networks, metabolic reaction networks
- World Wide Web
- Food Webs

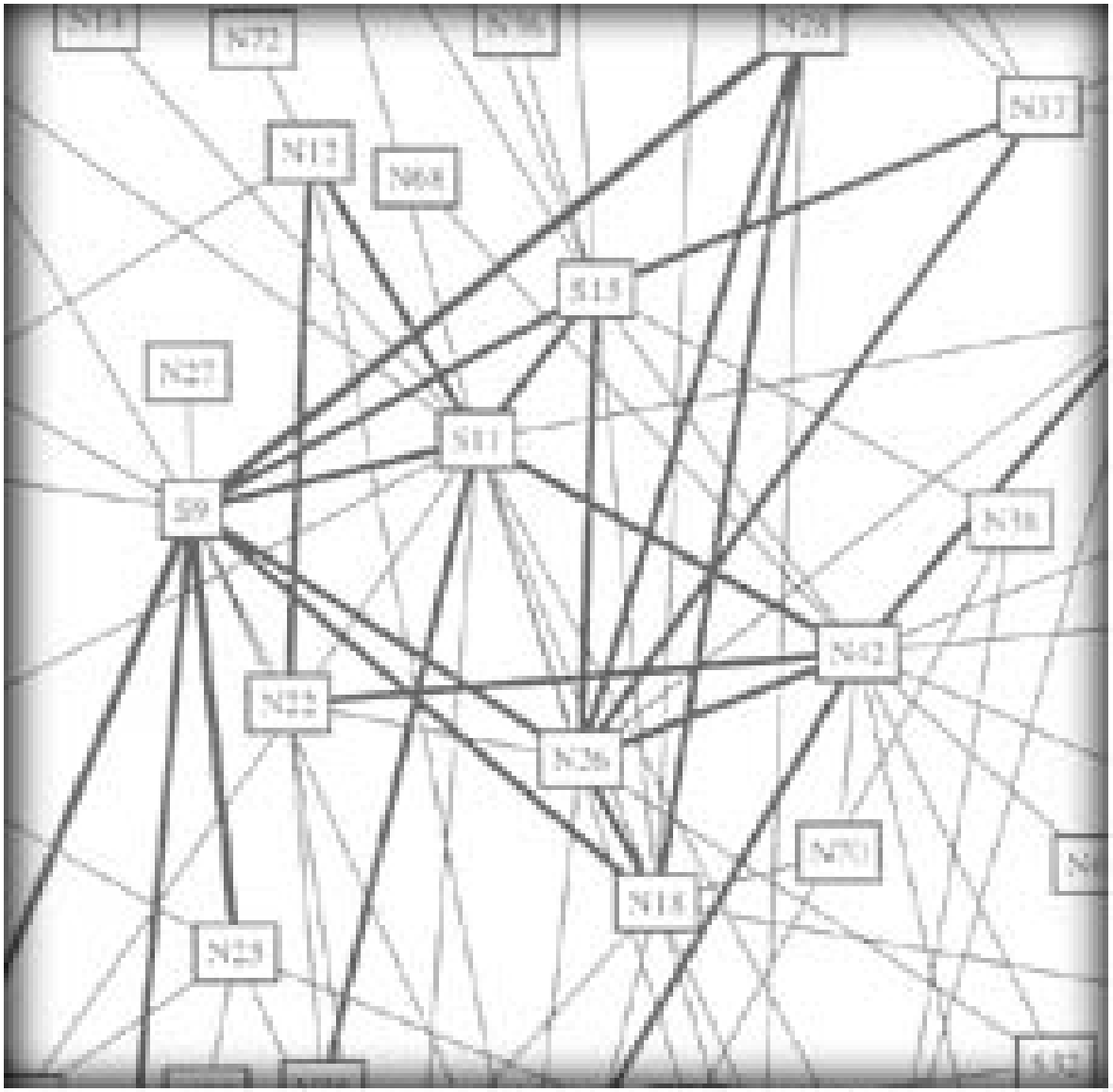


Online Social Relationships

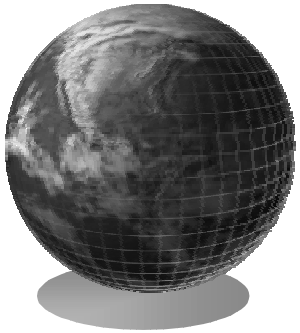


[Isbell et al.]

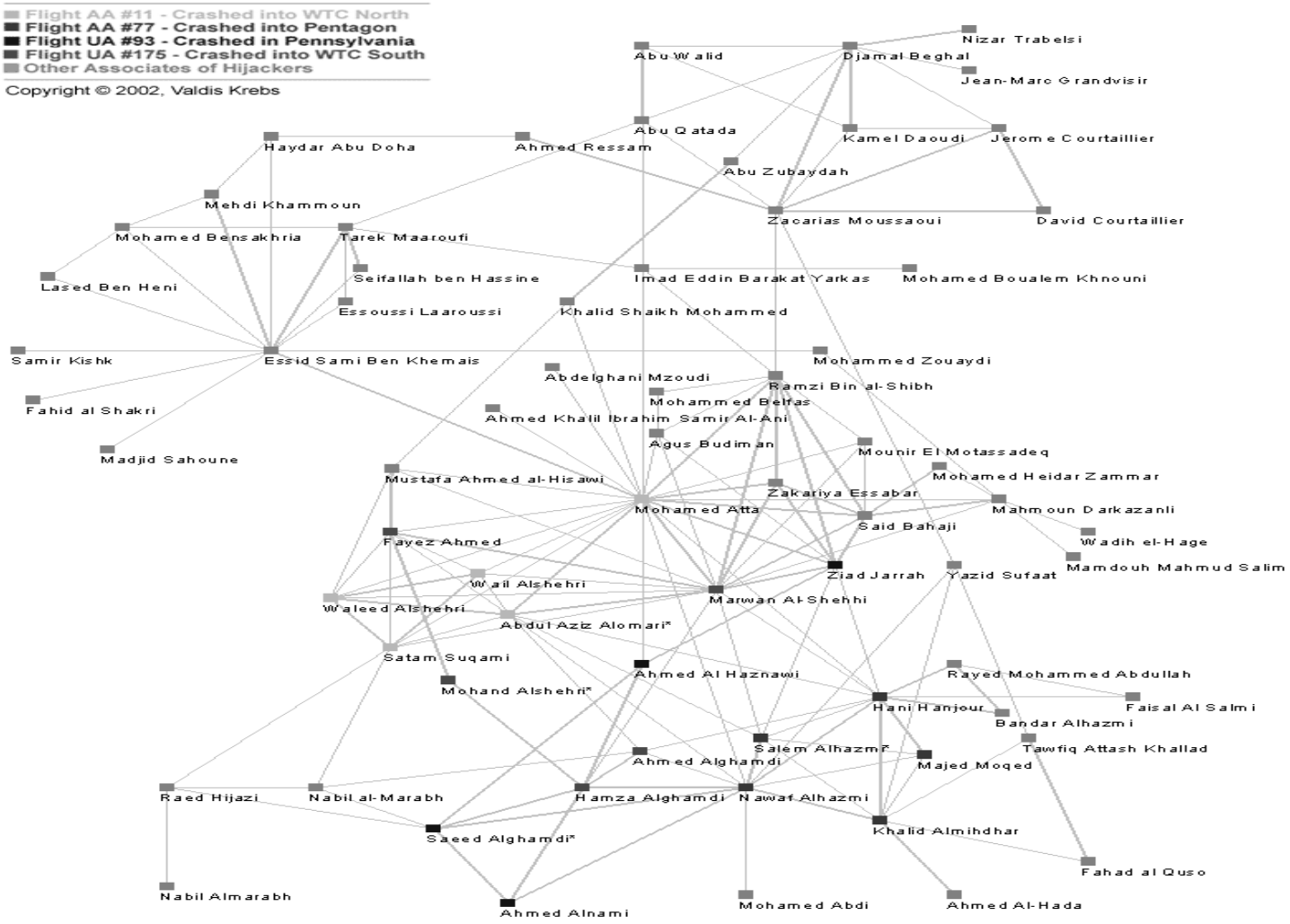
Syphilis transmission in Georgia

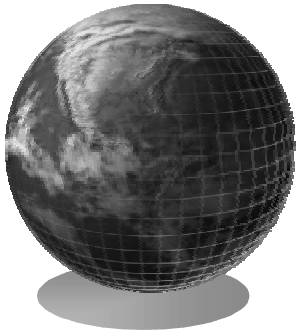


Network Visualization of the Outbreak

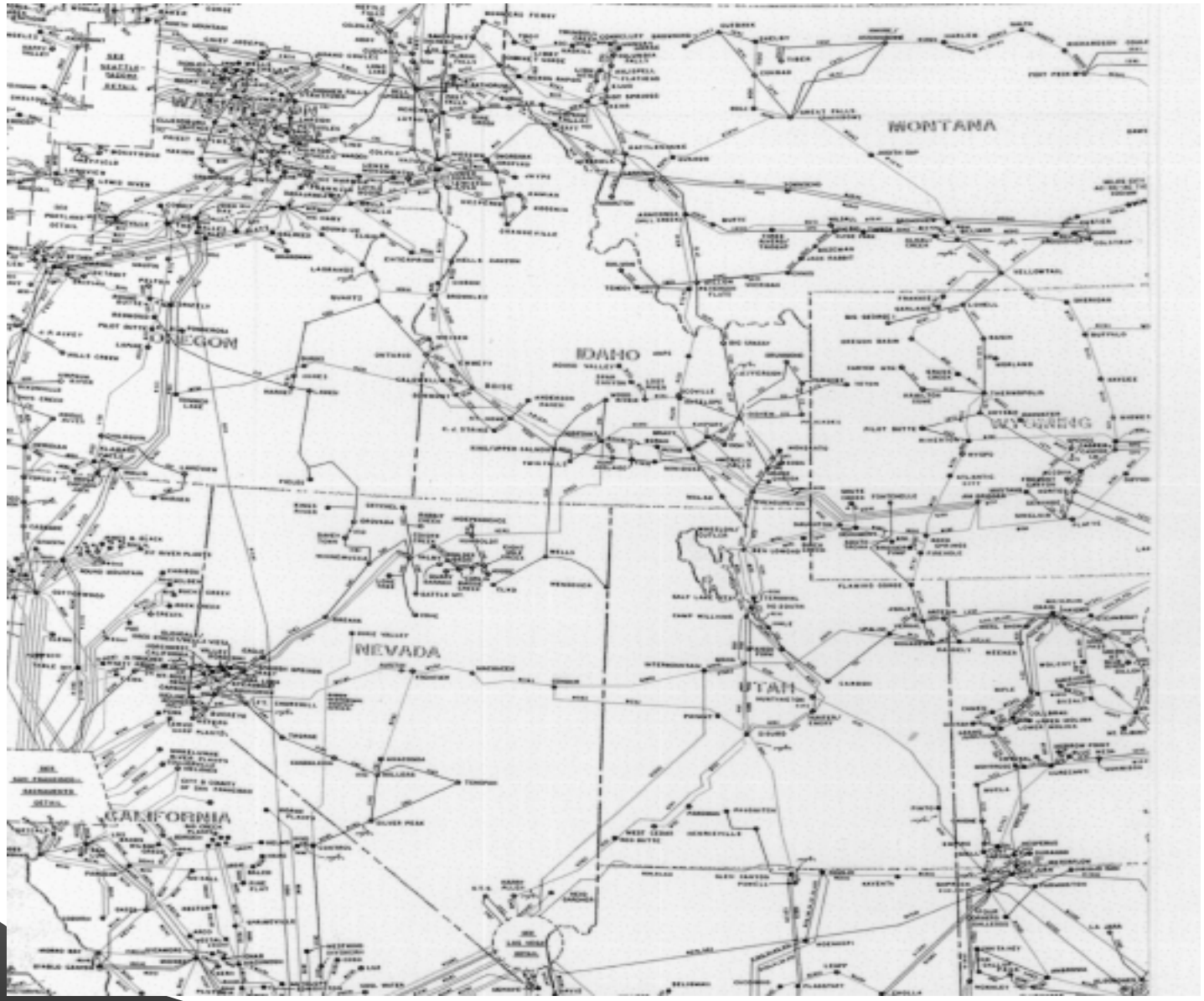


The Sept 11 Hijackers and their Associates





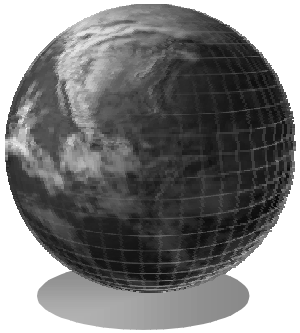
Power Transmission Grid of Western US



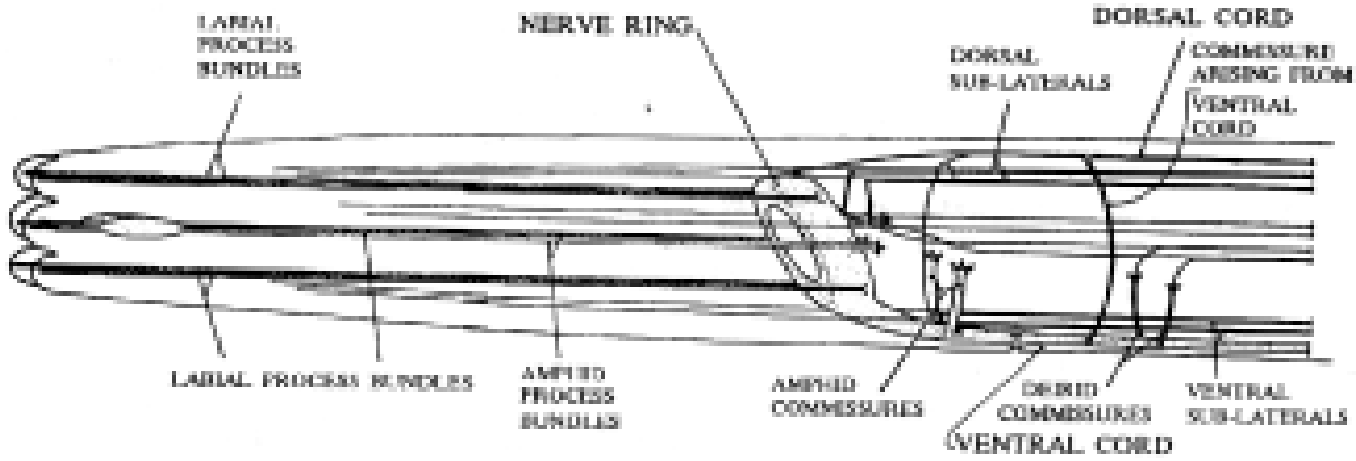
C. Elegans

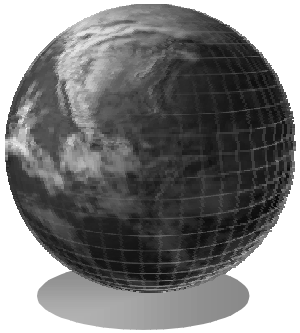

intestine
pharynx
testis
vas
deferens
cloaca
fan
spicules
rays


ovary
intestine
pharynx
oviduct
oocytes
uterus
eggs
vulva
rectum
anus



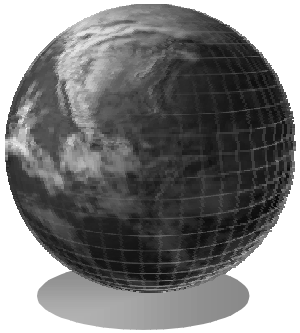
Neural network of *C. elegans*





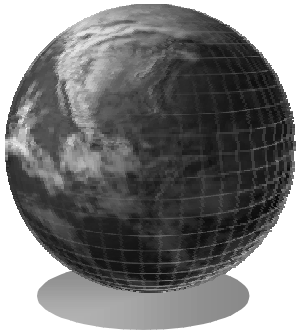
Six years later…

- We (collectively) have a good understanding of how the small world phenomenon works
- Also starting to understand other characteristics of large-scale networks
- New theories, better methods, faster computers, and electronic recording all contributing to rapid scientific advance



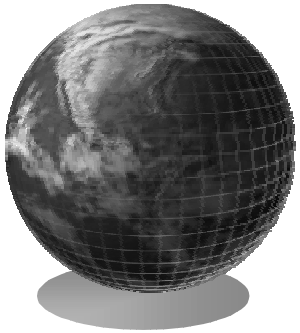
Social Search

- It's one thing for any two nodes in a network to be separated by only six degrees
- But it's quite another for people to be able to find the right 6
- Only have "local" information about the network
- Can't broadcast to everyone (virus)
- Instead must forward to just one acquaintance
- Call this *social search*



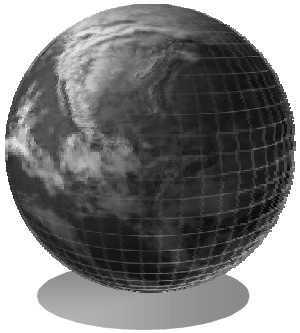
What did Milgram really show?

- Social search is a hard problem
- Obvious for similar people, but not obvious for very different people
- Evidence from Milgram's experiment limited to single target, and only 96 chains started with random sources in Omaha
- Yet the statement about "six degrees" is applied universally
- How universal is it?
- How does it work?



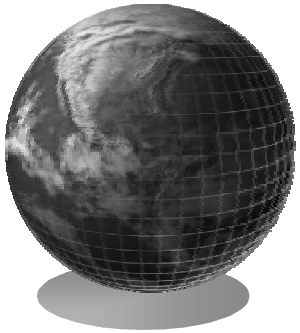
The Electronic Small World Project http://smallworld.columbia.edu

- 18 Targets
  - A university professor in upstate New York
  - A policeman in Perth, Australia
  - An librarian in Paris
  - A veterinarian in Norway, etc…
- 24,163 chains passing through 61,168 hands in 166 countries
- Name, location, occupation, gender, religion, SES status recorded for all participants
- Nature, origin and strength recorded for every tie



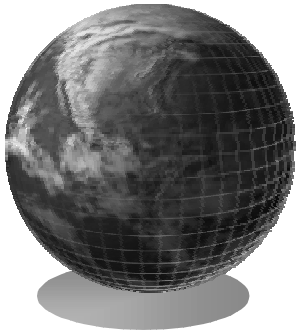
Results, I

- People tend to choose acquaintances of same gender as themselves (but also biased by gender of target)
- 96% of relationships offline (so not about technology)
- Friends most frequent choice, but most of the friendships arose in the workplace
- Most ties neither “weak” nor “strong”
- Geography and occupation dominate choices (geography for first few steps, then occupation)
- Number of friends apparently not important (not selected for, and no “funnels” near target.



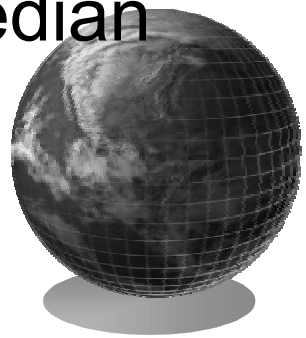
Results, II

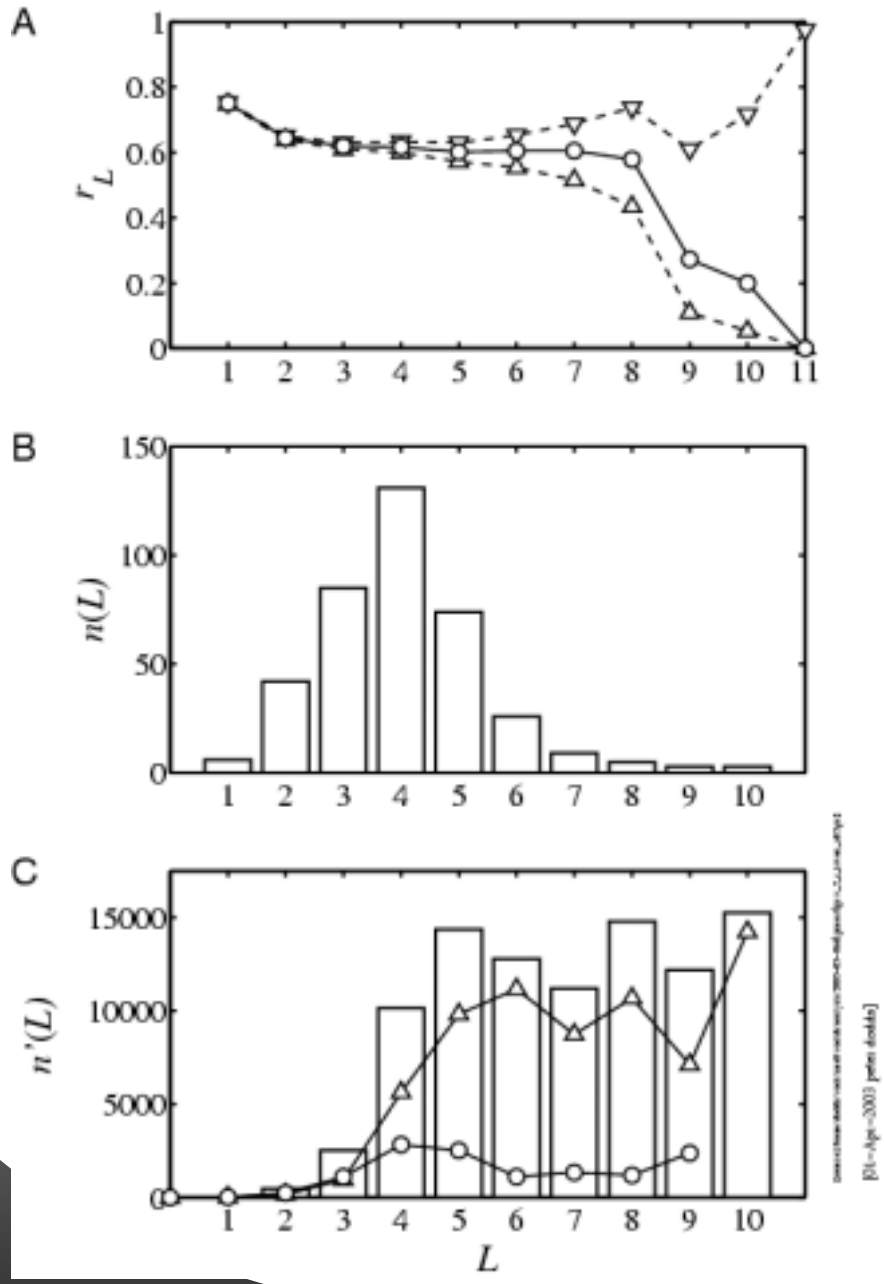
- Only 384 chains (1.6%) made it all the way to target
- 155 (40%) of these went to a single target (the university professor)
- Relative to unsuccessful chains, successful chains more likely to
  - Use professional ties
  - Use “weaker” ties
  - Not pay attention to number of friends



Results, III

- Average length of Observed Chains about 4
- Corresponds to Milgram's 6
- But both these numbers are misleading, because loss of chains due to attrition biases completions to shorter chains
- Assuming that chains terminate randomly, we can estimate what typical path length would have been with no attrition ("Ideal distribution")
- Find that if all chains had completed, median path would be
  - 5 if source and target in same country
  - 7 if source and target in different countries

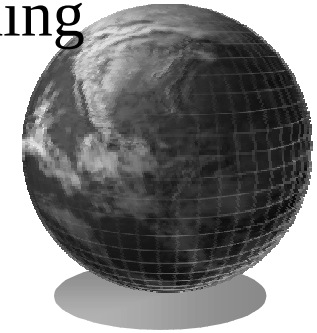




Attrition rate
versus
chain length

Distribution of
observed chain
lengths (Av=4)

"Ideal" Distribution
of chain lengths
(i.e. with zero attrition)
median L 5-7, depending
on separation of
source and target.

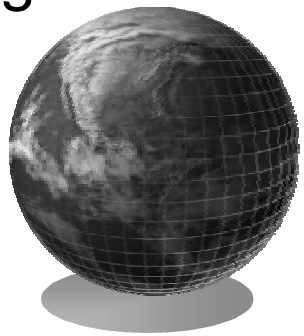


Why did so few chains complete?

- Response rate was 37% (really good!)
- But, equals Attrition rate of 63% *per link*
- Compounded over 6 links (1/3)^4 = 1/81, which is actually less than 1.6%
- Extreme loss of chains can be explained entirely by technical difficulties like limited incentive, motivation, etc.
- Entirely disconnected from question of how possible it is to find the target
- It is feature of the measurement instrument, not the network

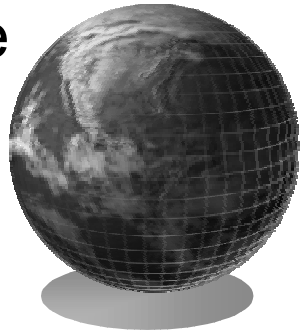
Why did one Target get 40% ??

- Target in question was the university professor
  - Superficially, he looks like a superstar
  - But he's an ordinary guy
- Main difference was his attrition rate per link was 10-15% lower than any other target!
  - Accounts for all his additional success
- Why?
  - Half our participants were in US, and many were college educated
- So he *seemed* easier to reach
  - Thus they were more inclined to participate
- Result is a self-fulfilling prophecy, but with a huge amplifier effect



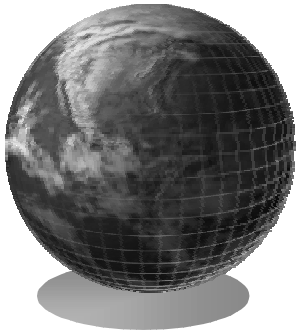
What does this tell us?

- Social networks are connected in “small-world” sense
- Social network also “searchable”
  - much harder to explain
  - The network enables “collective intelligence”
- But actual success depends *very sensitively* on incentives / motivation
  - 15% decrease in attrition over 6 steps, yields 800% increase in success rate
  - Reducing chain from 7 to 5 steps with same attrition (63%) yields factor of three increase
- Perception appears to play major role

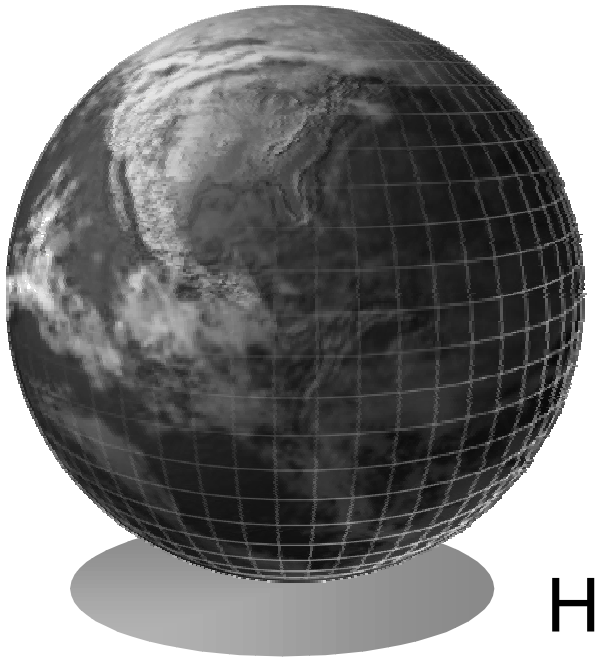


How does it help us?

- Shows us that experiments are necessary in order to understand social networks
  - Network structure clearly insufficient
- Learning from “social solutions” to hard problems can help technology
  - Distributed databases
  - Peer to peer networks
  - Next generation web searches
- Can inform our notions of social capital
  - Motivation matters more than access
- Can help us design better protocols for ambiguous problem solving



Six Degrees:
The Science of A Connected Age



Home Page

http://www.sociology.columbia.edu/people/index.html

Small World Project

http://smallworld.columbia.edu