The small-world model of Watts and Strogatz [1998]

A simple model to explain the small diameter of social, telephone, railroad, and other networks.

- Take a ring (circle) of n nodes in which every node is connected to the next k (say 2) nodes.
- Randomly re-wire each edge to a random destination with probability p.
- The resulting graph has logarithmic diameter with high probability:

$$diameter = O(\log n)$$

where $diameter = max_{u,v} distance(u, v)$

Kleinberg’s model

The web graph: Nodes=Pages, Edges=Hyperlinks

outline of the web graph

More findingw916 RG . 9916 RG th web-250(graph)]TJ ET1 0 0 1 -293.277 17.457

The big picture vs the local structure

- We can distinguish between global properties of the web graph and its local structure. Alap53ctur674(itur66(hastur674bween)-66(createdtur674(en)-66(can)-67

Hubs and authorities

- A

Power laws

What is a power law?

Two quantities y and x are related by a power law if

$$y \approx x^c$$

for some constant c.

log(x)

Brief history of power laws

Brief history of power laws (cont.)

Power laws and the Web

It has been observed that power laws appear in many aspects of the Internet and the Web. For example, the number of users per site obeys a power law.

Why such an abundance of power laws?

The Barabasi-Albert model [1999]

Albert and Barabasi proposed the following simple model for random graphs:

- Start with one node (or a small fixed graph)
- Add one-by-one nodes
- Each new node is connected to m nodes selected randomly with **probability proportional to their degree**.

The idea behind the model is simple: New pages tend to have links to "popular" pages.

The model is called the Barabasi-Albert model, or the preferential attachment model, or the rich-get-richer model, or the LCD model.

Results about the preferential attachment model

- Albert and Barabasi contacted experiments with their model and they found that the degrees obey a power law. They also, together with Jeong, gave a heuristic argument.
- Bollobás, Riordan, Spencer, and Tusnády [2001] proved (de)153e(Bolpo)25inged (

Preferential attachment graphs: diameter and clustering

The copying model

Power laws for the copying model

The Cooper-Frieze model

Cooper and Frieze proposed and analyzed a general model that has many pa-

Power laws for the Internet

The Internet graph: Nodes=Computers and Routers, Edges=links The degree distribution of the Internet seems to satisfy a power law. Here is the log-log plot of the frequency vs the outdegree from the Faloutsi paper.

$y(x$

Internet is not Web

The graphs of the Internet and the Web share some characteristics but differ completely on others.

Apparently the models for the Web (a virtual network) are not good for the Internet (a physical network). The most important difference is geography: A

The HOT model of Carlson and Doyle

The Fabrikant-K-Papadimitriou model (cont.)

The Fabrikant-K-Papadimitriou model (cont.)

In our model, node i attaches itself to the node j that minimizes the weighted sum of the two objectives:

$$\min_{j<i} \quad \alpha \cdot d_{ij} + h_j,$$

where α is a parameter that may depend on the final number n of points.

Results

The behavior of the modem6sependsmodem6crucially(vior)-n50(of)-250(the)value(vior

Experiments

Tree generated for $\alpha = 20$ and $n = 100,000$.

Explanation — Proof

So what? Why do we care about power laws?

Possible answers:

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