WATER RESOURCES RESEARCH, VOL. 31, NO. 4, PAGES 1119–1127, APRIL 1995

Can one gauge the shape of a basin?

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Abstract. This paper investigates the effects of geometrical factors characterizing the shape of a river basin on the features of its hydrologic response. In particular, we wonder if by measuring the hydrologic response (i.e., gauging) the salient geomorphic features of the basin can be recovered. We argue that the basic structure of the channel network tends, in ideal conditions, to yield some universal characters of the width function $W(x)$ defining the relative proportion of a contributing area at a distance x from the outlet. $W(x)$ exhibits low-frequency features, which are geometry-dominated, and high-frequency features determined by recurrent aggregation patterns. It is suggested that given the shape of the basin one can indeed forecast in a rational manner the main characters of the hydrologic response which are imprinted in reproducible width functions. However, the inverse problem (i.e., the determination of the shape from the measure of the hydrologic response) is less solidly defined because of the possible loss of irretrievable information induced by the dynamics of runoff processes. Therefore the question posed in the title cannot be solved in general, although many elements for a general theory are seemingly established.

Introduction

In a beautiful paper entitled "Can one hear the shape of a drum?," *Kac* [1966] wondered if someone with perfect pitch can determine the precise shape of a drum just by listening to its fundamental tone and all the overtones. This question identifies an inverse spectral problem and motivates numerous works on the subject. The question, far from trivial, is linked to the asymptotics of the infinite sequence of eigenvalues arising in the drum vibration context and to whether from such a sequence one can indeed bear complete inference of the geometric boundary conditions. Borrowing from Kac's title, revisited later in the context of fractal domains for the boundary value problem [*Lapidus*, 1989], this paper investigates the bearing of geometrical factors characterizing the shape of a river basin on the features of its hydrologic response. In particular, we wonder if by measuring the hydrologic response (i.e., gauging) the salient geomorphic features of the basin can be recovered in some manner. Conversely, we wonder if given the shape of the basin one can indeed forecast in a rational manner the main characters of the hydrologic response.

These questions, besides being far from trivial, bear important practical consequences to hydrology. In fact, flood prediction in ungauged basins still stands as a crucial engineering problem for disaster prevention. Since the shape of the basin can be obtained objectively, say from space, the link of shape and response is relevant to the problem since the knowledge of some geomorphic shape factors and spatial scales coupled with climatic observations could surrogate costly, and at times complex, measures.

The observation of recurrent fractal characters embedded in a variety of forms in river basin geomorphology is of relatively recent attention [e.g., *Mandelbrot*, 1983; *Tarboton et al.*, 1988; *La Barbera and Rosso*, 1989; *Rodriguez-Iturbe et al.*, 1992a, c, 1994; *Rinaldo et al.*, 1992, 1993; *Ijjasz-Vasquez et al.*, 1993, 1994]. The inference of the fractal characters of the basins on the hydrologic response (the so-called geomorphological dispersion) has been investigated by *Rinaldo et al.* [1991] and more recently by *Snell and Sivapalan* [1994]. *Rinaldo et al.* [1991] showed the travel time distribution to the outlet of the basin in response to a uniform instantaneous injection was solved analytically in the case of Wiener dynamics, that is, characterized by biased convection in the direction of the network to which a hydrodynamic dispersion process is superimposed. Interestingly, from a general expression of a moment-generating function, *Rinaldo et al.* [1991] inferred that the variance of travel times to the outlet is made up by two independent contributions: one related to the scatter of arrivals induced by hydrodynamic dispersion and the other, by far predominant in real cases, related to the geometrical heterogeneities, that is, to the variability of the connected paths to the outlet from a random starting site. It was concluded that the bulk of the response is geometry-dominated because geomorphological dispersion tends to prevail over hydrodynamic dispersion. Interestingly though, they found that dispersion mechanisms tend to destroy fractal characters in the structure of the hydrologic response.

From the perspective of science the link between fractal forms and the dynamics behind their growth is gaining strength, especially after the discovery of self-organized criticality [*Bak et al.*, 1987, 1988] (for a specific reference to geomorphology, see also *Bak and Paczuski* [1993]). In that milieu, river networks have played an important role both for the availability of reliable data from the real world covering several orders of magnitude in their spatial scales and for the recognition of the signatures of self-organization in their fractal structure [*Rinaldo et al.*, 1993; *Rigon et al.*, 1994; *Rodriguez-*

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Paper number 94WR03290.
0043-1397/95/94WR-03290$05.00

Iturbe et al., 1994; A. Rinaldo et al., Climatic signatures on geomorphology, unpublished manuscript, 1994 (hereinafter referred to as Rinaldo et al., unpublished manuscript, 1994)].

The geometrical and topological connections of the hydrologic response were first suggested by *Kirkby* [1976], although a complete formalization in the framework of the theories of transport by travel time distributions came only after the theory of the geomorphologic unit hydrograph (GIUH) [*Rodriguez-Iturbe and Valdes*, 1979]. The so-called width function formulation of the GIUH by *Gupta et al.* [1986] (see also *Gupta and Mesa* [1988]) later extended the geomorphic insight of the theory. The basic result for the response in time of a basin (identified by the travel time distribution $f(t)$) to an instantaneous unit impulse of effective rainfall uniformly distributed in space is as follows:

$$f(t) = [uW(ut)]/l \tag{1}$$

where t is time, in suitable units; u is a velocity, constant for all particles, defining a dynamic scale for the process (here a sample water particle undergoes a constant drift u along the local direction of the links of the network); l is a geometrical scale, here the maximum length from source to outlet; and $W(x)$ is the (dimensionless) width function of the basin, that is, the relative proportion of basin area at a distance x from the outlet. Notice that the length x is measured along the network (i.e., the chemical distance) and that $W(x)$ is actually an area distribution function. Since no distinction is drawn at this stage between hillslopes and channels, the two functions coincide. Interestingly, one would expect that the observation of recurrent multifractal characters for $W(x)$ [*Marani et al.*, 1991; *Rinaldo et al.*, 1992, 1993; *Rodriguez-Iturbe et al.*, 1992b] would be transfered through (1) directly into the hydrologic response. It is now clear [*Rinaldo et al.*, 1993; *Marani et al.*, 1994] that the width functions $W(x)$ of real basins reflect both common recurrent characters (independent of climate, geology, vegetation, and the shape of the basin's boundaries) and some features peculiar to the shape of the basin. For instance, the power spectrum of the width function clearly shows that low-frequency modes are determined by the outer shape of the basin, whereas high-frequency modes tend to a power law with universal slope. Although the power spectrum hinders multiscaling characters which may be revealed by other analyses [e.g., *Feder*, 1988], this general result is confirmed through different degrees of sophistication [*Rodriguez-Iturbe et al.*, 1994; *Marani et al.*, 1994].

The result in (1) was generalized by *Marani et al.* [1991] as follows:

$$f(t) = [u\langle W(ut)\rangle]/l \tag{2}$$

where $\langle W(ut)\rangle = \int_0^l W(x)\, dP(x, t)$ where P is the solution to Kolmogorov's backward equation: $dP(x, t) = dx/(4\pi Dt^3)^{1/2} \exp[-(x - ut)^2/4Dt]$. The above model postulates that, in addition to a constant drift u, the dynamics of any water particle are affected by a one-dimensional Brownian motion $X_B(t)$ biased in the direction of the network ($\langle X_B\rangle = 0$ and $\langle X_B^2\rangle = 2Dt$). Similar results were derived in different contexts [*Troutman and Karlinger*, 1985]. The result in (2) yields important contents of information. In the general case one can derive analytically the moments of the distribution $f(t)$ and in particular the mean and variance of the arrival time distribution [*Rinaldo et al.*, 1991]:

$$E[T] = \sum_{x=1}^{l} W(x)\,\frac{x}{u} \tag{3}$$

(where l indexes the maximum length from source to outlet along the network) and

$$\mathrm{Var}[T] = 2\sum_{x=1}^{l} W(x)\,\frac{xD}{u^3} + \sum_{x=1}^{l} W(x)\left(\frac{x}{u}\right)^2 - \left[\sum_{x=1}^{l} W(x)\,\frac{x}{u}\right]^2 \tag{4}$$

We note that the above equations hold even in the case where velocity depends on the position, $u(x)$, with obvious simplification in the case of constant drift u. We also note that the morphological contribution to the variance of the hydrologic response (the second and third terms in (4)) is null only in very particular cases.

The following was inferred from (2):

1. Fractality of basin shapes (i.e., without a ruler one cannot distinguish networks extracted from very large or small basins) cannot be transferred to the dynamic response $f(t)$. In fact, as a basin Peclet number $Pe = ul/D$ decreases (i.e., when mean travel distance is large with respect to mean dispersion displacements), the multiscaling self-affinity observed for real width functions [*Rinaldo et al.*, 1992, 1993; *Marani et al.*, 1994] is progressively destroyed. Thus one can distinguish from the hydrologic response whether the basin is large or not simply from the regularity of the gauged record (the larger the basin the smoother the gauge trace). Therefore the mechanisms and the timescale imposed by dispersion break the one-to-one relationship of width functions and unit hydrologic responses.

2. While it is observed experimentally that the width function is not always skewed to the left with positive third moments of the distribution, being in some cases (e.g., Peano's basin) indeed the opposite, the hydrologic response is invariably skewed to the left, showing a relatively fast rising limb and a long tail. It was argued that the latter is described analytically by (2) because dispersion has a selective and pronounced skewing effect.

However, the above analyses were missing some important parts of the picture related chiefly to hillslope patterns and heterogeneity which are studied here. In this paper we will focus on direct runoff, therefore bypassing the estimation of effective rainfall. Although this can be dealt within the framework of the geomorphologic hydrologic response, we prefer to avoid clouding the central idea of this paper with a different kind of problem. Also, we neglect spatial nonuniformities of rainfall input which could be accounted for by the geomorphological approach. In other words, we are not interested in the size of the basin and of the input, as much as *Kac* [1966] was not interested in what was forcing the drum to vibrate. The role of the sound in the work by Kac will be taken by the geomorphologic unit hydrograph here.

On the Interplay of Width and Boundaries

In this section we show that the information contained in the width function allows for decoding the main features of boundary profiles of river basins.

We have mentioned in the introduction that the geomorphic

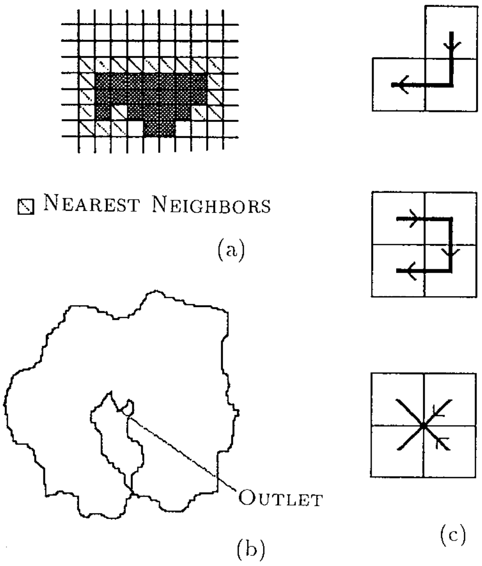

Figure 1. (a) The sequence of possible pixels (i.e., at a given distance x from the seed) is chosen among the neighbors of the previously occupied sites. This is done through adjacent layers preventing the formation of pits or holes in the drainage structure; (b) one example of an unphysical configuration obtained by the procedure and then discarded (the structure resulting from a sequence of additions merges leaving a hole surrounding the outlet). In the scheme adopted, the choice of pixels not belonging to an arbitrary region seeded in the outlet is constrained to avoid unphysical configurations. In practice, one rejects a realization in which the random choice selects additions whose coordinates belong to an unfeasible predefined region. However, this is not strictly necessary once ensemble averaging over a large number of realizations is employed; (c) unphysical choices of drainage directions are also avoided [*Groff*, 1992].

width function $W(x)$ has recurrent characters reflecting common mechanisms of organization of the drainage structure regardless of climate, geology, vegetation, etc. These recurrent characters pertain to the high frequencies of other spectra. The low frequencies are much more variable from basin to basin as they reflect the bulk of the contributing area that is available to drainage. Therefore the low frequencies are grossly the width of the basin from ridge to ridge measured along a direction orthogonal to a diameter. Needless to say, at a higher scale this availability of drainage area, as well as the overall fractal shape of the boundaries, is regulated by the competition for drainage and the migration of divides produced by landscape self-organization. Within this context we assume that the boundaries are assigned and time-invariant.

Although the inverse nature of the problem was speculated to yield nonunique solutions, we have investigated under what circumstances the identification results are meaningful. One such example may be obtained by a simple exercise in which one constructs a network by successive random additions from a seed which respect the total number of occupied sites among all available nearest neighbors. The random additions have to be exactly equal to the width function which is given a priori. Although there is no unique tree respecting this rule, the main characters are reproduced as follows:

1. A square lattice equal to the digital elevation model (DEM) lattice size that allowed the extraction of the given width function is assumed. The arbitrary coordinates of the outlet are assigned, say (0, 0).
2. All possible neighbors to the outlet site, that is, at a distance $x = \Delta x$ (where Δx is the grid size), are isolated in the plan. We have constrained the possible choices based on physical requirements as discussed below.
3. The sequence of possible pixels (i.e., at a given distance x from the seed) that might be chosen at the next level from a given structure is defined. This is done through adjacent layers (Figure 1a) which prevent the formation of pits or holes in the drainage structure. From that sequence we eliminate all sites already occupied by the growth. We also eliminate pixels not belonging to an arbitrary half-space through the seed to avoid unphysical configurations (Figure 1b). Notice that particular choices of drainage directions (Figure 1c) are avoided because they are unphysical.
4. Among all possible M choices of neighbors, a subset is chosen to allow for the correct proportion $W(x)$ of pixels among the neighbors. The choice of neighboring elements identifies the drainage directions joining the pixels. Among the M possible choices of pixels equally distant x from the outlet, the choice is made by drawing $W(x)N$ pixels randomly from a uniform distribution, labeling all pixels that may be chosen and avoiding discarded sites. If the available pixels are less than those required by $W(x)$, the configuration is discarded. Notice that through the above procedure it is highly unlikely that regular shapes will be obtained (e.g., mirror-image basins).
5. We set $x = x + \Delta x$ and reset the procedure until $x = l$.

The development of the structure of the basin follows the availability of area ruled by $W(x)$. Thus the final choice is the final step of a sequence of compatibility controls on the resulting structure. Through the above procedure any realization of the random process is characterized by the prescribed width function $W(x)$. We have also tested other strategies without significant changes.

Figure 2 shows the boundaries and the width function of the Nelk river basin, extracted by a 62.6 $m^2 \times$ 92.2 m^2 DEM [*Tarboton et al.*, 1988; *Ijjasz-Vasquez et al.*, 1993]. Figures 3a and 3b show two realizations of the boundaries of the structure resulting from the above procedure once the width function of Nelk is imposed. Figure 3c shows the reconstructed shape as

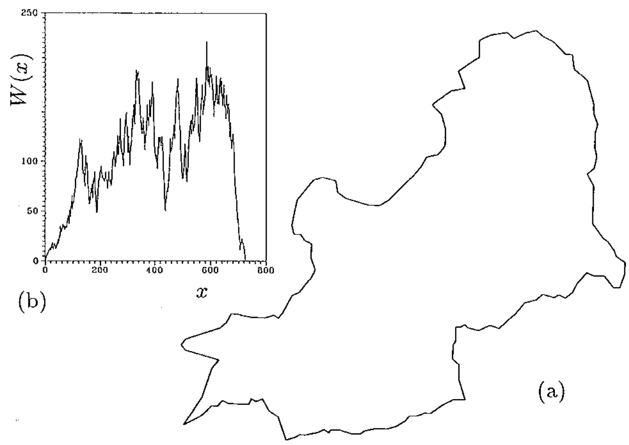

Figure 2. (a) The boundaries and (b) the width function $W(x)$ (in pixel units) of the Nelk river basin (440 km^2, extracted by a 62.6 $m^2 \times$ 92.2 m^2 digital elevation model (DEM) [*Tarboton et al.*, 1988; *Ijjasz-Vasquez et al.*, 1993]).

Figure 3. (a, b) Two realizations of the boundaries of the structure resulting from the procedure described here. The width function of the Nelk river basin is imposed and is exactly reproduced in all realizations; (c) the reconstructed shape as the ensemble average of 10 realizations of the shapes devised according to the approach described here.

the ensemble average of 10 realizations of the shapes devised according to the previous elementary approach. The ensemble average is obtained by (1) unrolling the boundary profiles via a polar plot and expressing the resulting function as a Fourier series [*Russ*, 1994]; and (2) reconstructing the boundaries from the ensemble-averaged components of the Fourier spectrum of the polar plot.

The robustness of the resulting configurations of the basin boundaries is surprising. In fact, in almost all representations the overall shape of Nelk is reproduced (Figure 3). An objec-

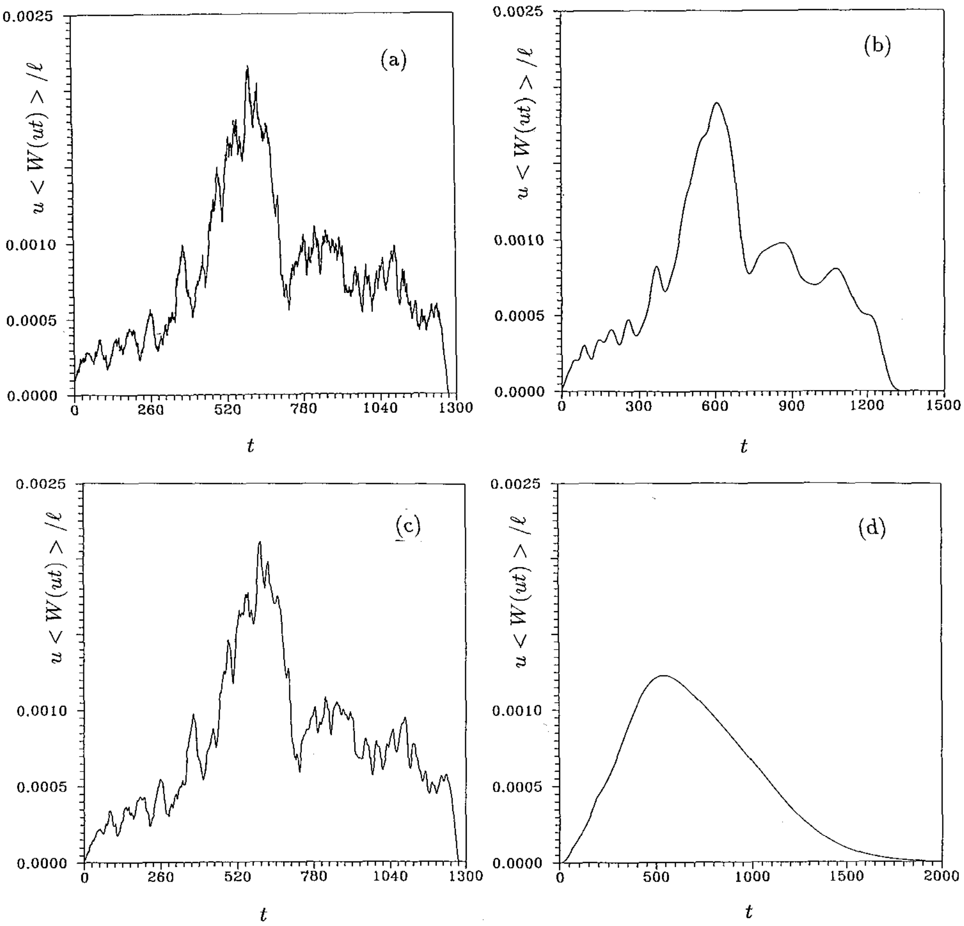

Figure 4. The width function of the Schoharie river basin (2408 km^2 [*Tarboton et al.*, 1988]) and the width function formulation (2) of the hydrologic response (in all cases, $u = 1$ and $l = 1$): (a) $D = 10^{-6}$ m^2/s; (b) $D = 10^{-2}$ m^2/s; (c) $D = 10^{-4}$ m^2/s; (d) $D = 1$ m^2/s.

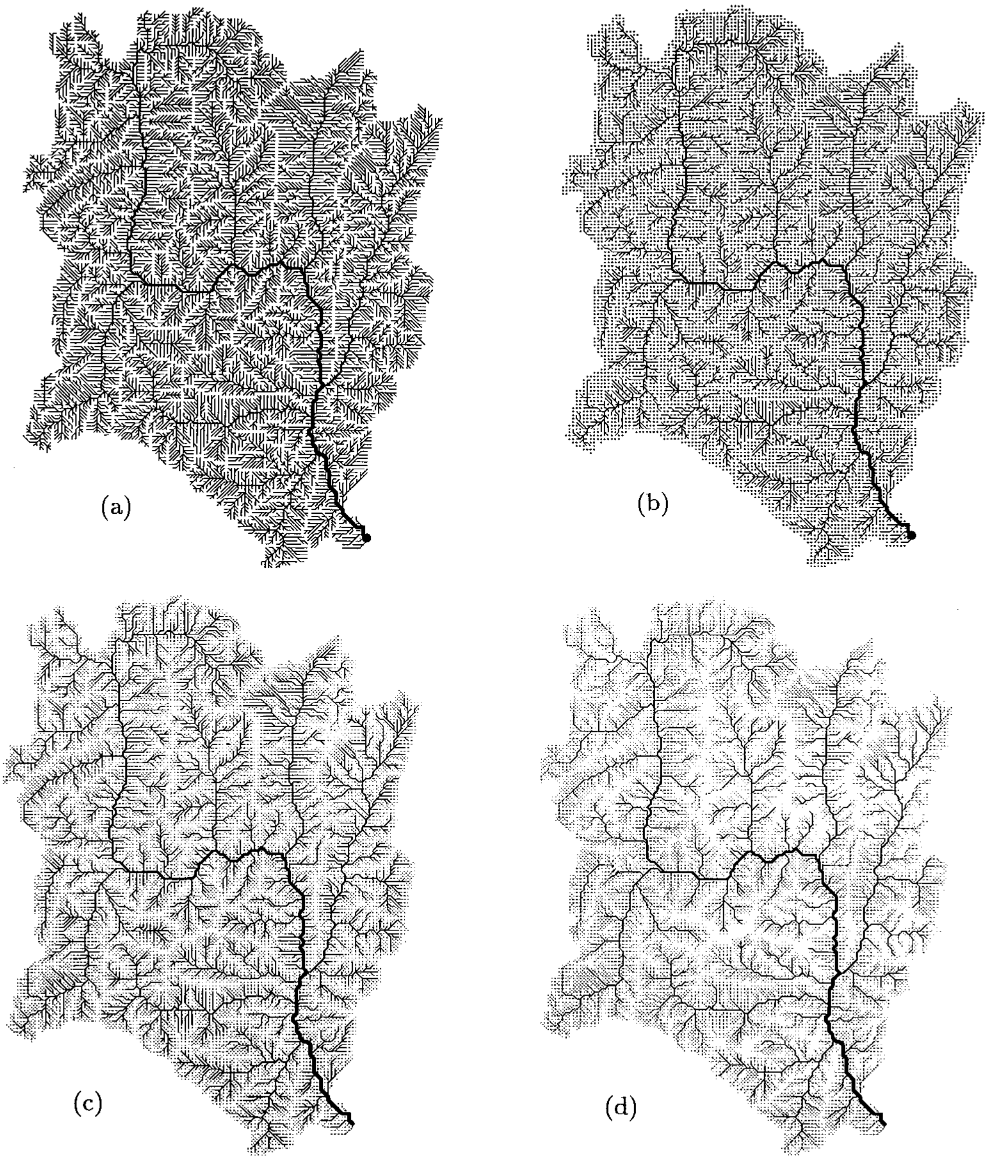

Figure 5. (a) Drainage directions for the DEM of the Fella river basin; (b) channel network identified by connecting all concave sites for the Fella river basin in Figure 5a; (c) Fella river network. The channelized site i has $\nabla^2 z_i \geq 0$ and $\nabla z_i (A_i)^{1/2} \geq 100$; (d) Fella river network. The channelized site i has $\nabla^2 z_i \geq 0$ and $\nabla z_i (A_i)^{1/2} \geq 400$.

tive boundary representation of the different results (i.e., a comparison of the amplitudes in the Fourier spectra of the polar plots of the boundaries [*Russ*, 1994]) substantiates the visual impression. This will be reported elsewhere (Rodriguez-Iturbe et al., manuscript in preparation, 1995).

We have also tried to simulate the same shape by using a filtered version of $W(x)$ where only low frequencies had been retained. The same robustness is shown by the shape of the basin. Here our interest is not in refined representations of the features of the boundaries (e.g., they could be modeled by two colliding self-affine trails issuing from the outlet [*Ijjasz-Vasquez et al.*, 1994]) in any realization, nor in evaluating ensemble-averaging properties, and thus we stop with the conclusion that the information contained in the low frequencies of the width function suffices in reconstructing the shape of the basin.

The possibility to suitably reproduce the width function from the knowledge of the outer shape of the basin had been noticed earlier [*Rigon et al.*, 1993] using optimal channel networks (OCNs) concepts [*Rodriguez-Iturbe et al.*, 1992a, b]. It had been established that OCNs developed with given basin boundaries have similar width functions among themselves and among those observed in real basins with the same shape. Since the link of width functions to the hydrologic response had also been established [*Kirkby*, 1976; *Gupta et al.*, 1986; *Gupta and Mesa*, 1988; *Rinaldo et al.*, 1991], the above result suggests that by gauging the flow at the control section of a watershed one could pursue the inverse pattern, from the flow to the width function and from the widths to the boundaries to effectively gauge the shape of a basin. This is the subject of the next section of this paper.

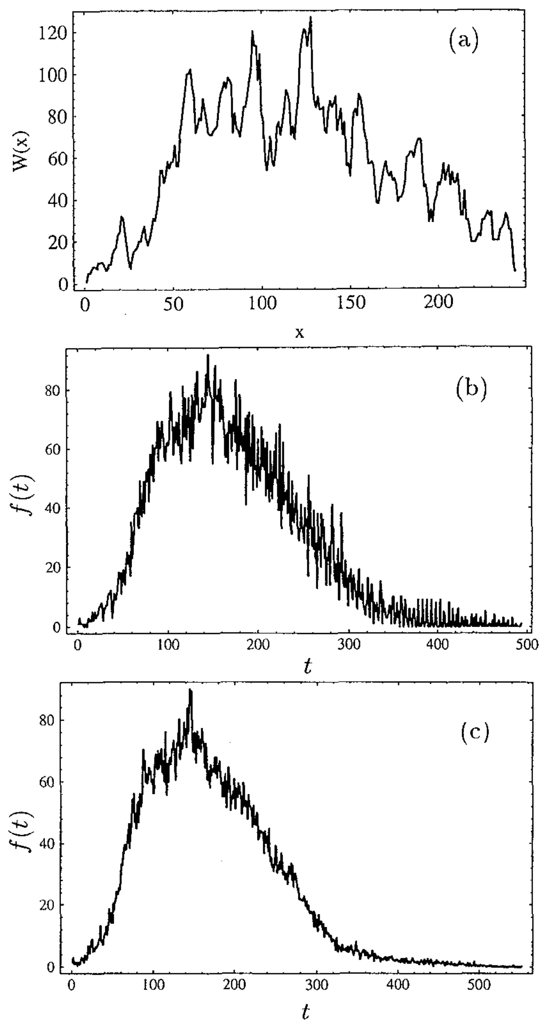

Figure 6. The hydrologic response obtained by sampling the travel times to the outlet for all sites i in the Fella river DEM of Figure 7: (a) the width function $W(x)$ (the particular case where $u_h = u_c = u = 1$ and $l = 1$ when $f(t) = W(ut)$); (b) the response $f(t)$ for the deterministic case where $u_h/u_c = 0.1$ and $u_c = 1$, the latter in pixel units; (c) the results of a Monte Carlo (MC) procedure where the drift u_h is randomized by assuming a lognormal distribution with mean 0.1 and variance 0.1. The curve is obtained as the ensemble average of five realizations.

Geomorphologic Hydrologic Response

Figure 4a shows the width functions of the Schoharie river basin (1412 km^2) whose DEM features are discussed by *Tarboton et al.* [1988]. In there as many as 100,000 pixels define the relative distribution of drainage area along the maximum path from source to outlet. Figures 4b–4d show the effects of introducing a hydrodynamic dispersion of the type contained in (2) superposed to a unit drift u applied to the width functions of the Schoharie river basin. The distribution of the arrivals tends toward a skewed form typical of a sigmoid hydrologic response. The appearance of a skewness in Figure 4 not contained in the original width function (Figure 4a) and the smoothing effect of the dispersion suggest [*Marani et al.*, 1991; *Rinaldo et al.*, 1991] that the pronounced skewness of real-life hydrographs could be based solely on dynamical arguments. It was also suggested that (2) explains why basins of different sizes tend to exhibit scale invariance in their geometrical features unlike their responses. In fact, the larger the basin, the smoother the response hydrographs tend to be. This follows from the embedded timescale and spatial scale imposed by dispersion processes.

The above suggestions were incomplete because the processes involved neglecting the role of hillslope patterns and heterogeneities. It turns out that these are of foremost importance.

To describe the former, one needs to objectively distinguish hillslopes, valleys, and channels. Hillslopes are seen as areas of topographic divergence, and valleys are areas of topographic convergence. Channels appear within areas of topographic convergence but are not defined by curvature alone. In fact, it is difficult to define a channel from DEMs, a morphologic feature defined by a concentration of transport of water and sediment within a defined geometry where, for instance, banks are meaningful concepts. However, one feature that can be objectively measured from accurate DEM data is the convergence (or divergence) of topography at any point of that landscape. In fact (let $z(\mathbf{x}) = z(x, y)$ be the field of landscape elevations), one can measure regions of convergent topography by the condition that

$$\nabla^2 z(\mathbf{x}, t) \geq 0 \tag{5}$$

(where ∇^2 is the Laplace operator ($\nabla^2 = \partial/\partial x^2 + \partial/\partial y^2$)) and vice versa ($\nabla^2 z < 0$) for areas of divergent topography. Examples of spatial patterns of convergent, divergent, and planar topographic elements compared to modeling of DEMs have been discussed by *Dietrich et al.* [1993], while *Howard* [1994] used critical values of the gradient divergence to discriminate channel initiation. The classification is based on the sign of $\nabla^2 z$ for discriminating between convergent or divergent elements. Elements are considered planar when $|\nabla^2 z| < \epsilon$ with ϵ as a suitable cutoff. The accurate evaluation of the Laplacian is an interesting numerical problem. However, in assessing the convergent or divergent nature of the topography, one is interested in the sign of $\nabla^2 z$, and thus the accurate evaluation of its absolute value is of lesser importance.

Divergent topographies require specific techniques since multiple flow directions ought to be assigned to pixels placed in a divergent landscape [*Costa-Cabral and Burges*, 1994; *Montgomery and Foufoula-Georgiou*, 1993] where the cumulated flow at a point (surrogated by the upslope area) should be distributed among more than one downslope neighbor. However, in the context of this paper we do not need to further discuss this point.

Thresholds for channelization also play an important role [*Montgomery and Dietrich*, 1988, 1992; *Dietrich et al.*, 1992, 1993]. They can be thought of as the by-product of subsurface saturation, slope instability through shallow landsliding, and/or erosion by overland flow. In the case of erosion by overland flow the threshold is clearly related to the exceedance of a critical shear stress. The actual shear stress at the ith site, say τ_i, is proportional to the support area and the local slope at i, say $\tau_i \propto (A_i)^{1/2}|\nabla z_i|$. Thus the exceedance of a critical stress ($\tau_i > \tau_c$) at the area necessary to support a channel head generated by the above mechanism ($A_i = A_t$) can be cast in the form $A_t = C/|\nabla z|^2$, where $C = f(\tau_c, p)$, p being some meaningful net rainfall intensity. Thus smaller drainage areas are needed to initiate channels on steeper slopes. Slope-dependent area thresholds A_t define quite well the real extent and structure of the channel networks from topographic data, and, according to *Di-*

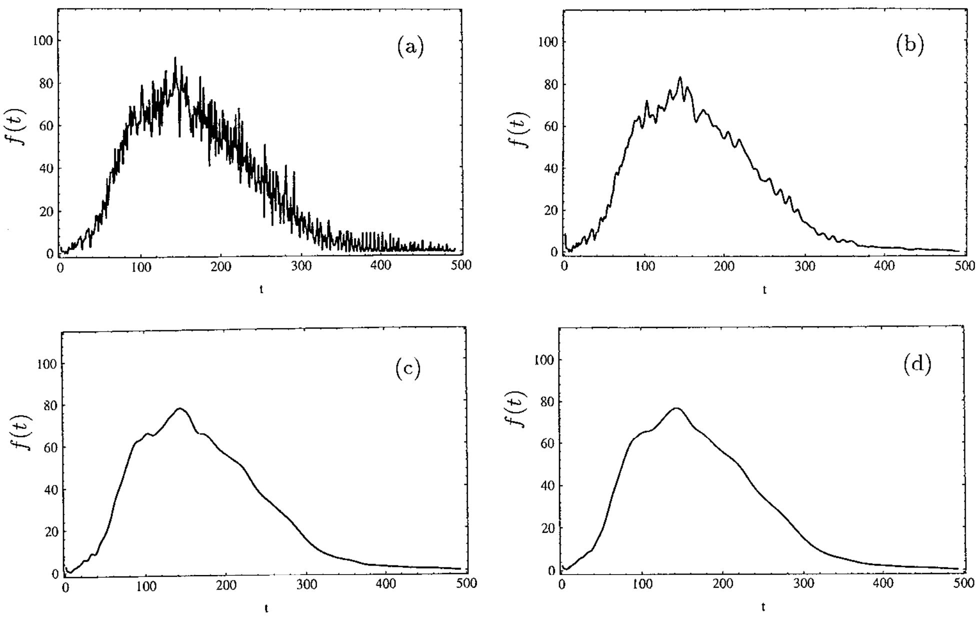

Figure 7. The effects of hydrodynamic dispersion on the responses of Figure 6. Here the dispersive process is modeled by (2) [*Rinaldo et al.*, 1991]. The dispersion coefficients have been set equal to (a) $D = 0$; (b) $D = 0.05$; (c) $D = 0.1$; and (d) $D = 0.2$ in pixel units (L^2/T). Notice that no MC average has been implemented.

etrich et al. [1993], the channelized network changes little even for relatively large changes in the value of τ_c.

Figure 5a shows a representation of the Fella river basin (northern Italy, 1420 km^2) whose DEM features are described by *Rodriguez-Iturbe et al.* [1994]. The map is obtained by associating a drainage direction of steepest descent with every pixel regardless of its channelization. We have extracted the channel network by defining the arbitrary channelized site i through the simultaneous occurrence of (1) convergent topography, that is, $\nabla^2 z_i \geq 0$; and (2) the exceedance of the critical threshold, that is, $\nabla z_i(A_i)^{1/2} \geq \tau_c$ (where z_i is the elevation at the ith site and A_i its cumulated area computed through the drainage directions). Figure 5b shows the network obtained connecting all concave sites. The related valley density (the relative proportion of concave pixels whose characterization is meaningful (Rinaldo et al., unpublished manuscript, 1994) is 0.62. Figures 5c and 5d show the results of the application of the threshold criterion to the Fella river basin with $\tau_c = 100$ and 400 in pixel units. Notice that drainage directions in unchanneled sites are represented by dotted lines. Notice also that notwithstanding the rather large range of values of τ_c, the channelized network undergoes minor changes.

With the above characterization one can objectively compute, for any path originated at site i, a measure of the hillslope paths $L_h(i)$ required to reach the first channel site, say j_i, and the related length of channel path $L_c(j_i)$ from j_i to the outlet, in both cases by following local drainage directions defined by ∇z.

Following the idea of *Van der Tak and Bras* [1990], we have rescaled the width function by suitably distinguishing the mean drift depending on whether the moving particle is in hillslopes or channels. Let u_h denote the mean drift in hillslopes and u_c the analogous for channels. Mean travel time T_i for the particle issuing at site i is therefore

$$T_i = \{[L_h(i)]/u_h\} + \{[L_c(j_i)]/u_c\} \qquad (6)$$

and the distribution $f(t)$ follows from sampling all sites i within a DEM. Notice that in this case the features of the distribution are completely imprinted in the morphology of the flow paths.

Figures 6a to 6c show the result of the above rescaling applied to the Fella river basin shown in Figure 5d. Figure 6a shows the width function $W(x)$ (obviously the particular case of response $f(t) = W(ut)$, where $u_h = u_c = u = 1$. Here $l = 1$ for convenience), which is not particularly skewed. Figure 6b shows (for the case where the threshold is as in Figure 5c) the response function computed by an arbitrary ratio $u_h/u_c = 0.1$ and $u_c = 1$, the latter in pixel units. Figure 6c shows the results when a Monte Carlo (MC) procedure has been applied to the process to mimic the effects of heterogeneities. In Figure 6c the drift u_h has been randomized by assuming a lognormal distribution with mean 0.1 and variance 0.1, and five realizations have been ensemble averaged.

From the results in Figure 6 it is clearly suggested that the different dynamic characterizations for hillslopes and channels, even in the gross deterministic framework of Figure 6b, drastically enhance the positive skewness of the response. The unphysical degree of roughness of the response of Figure 6b is significantly reduced by simulating heterogeneity through Monte Carlo techniques. It is not uncommon to find width functions with negative skewness appearance which the above mixed dynamics translate into a smooth response with a positive skewness.

Figures 7a–7d show the addition of hydrodynamic dispersion of the type described by (2) to the response in Figure 6b, where the related dispersion coefficients have been arbitrarily set to $D = 0, 0.05, 0.1$, and 0.2 in pixel units (L^2/T). Notice that no Monte Carlo average has been implemented.

The effects of progressive MC averaging are tested in

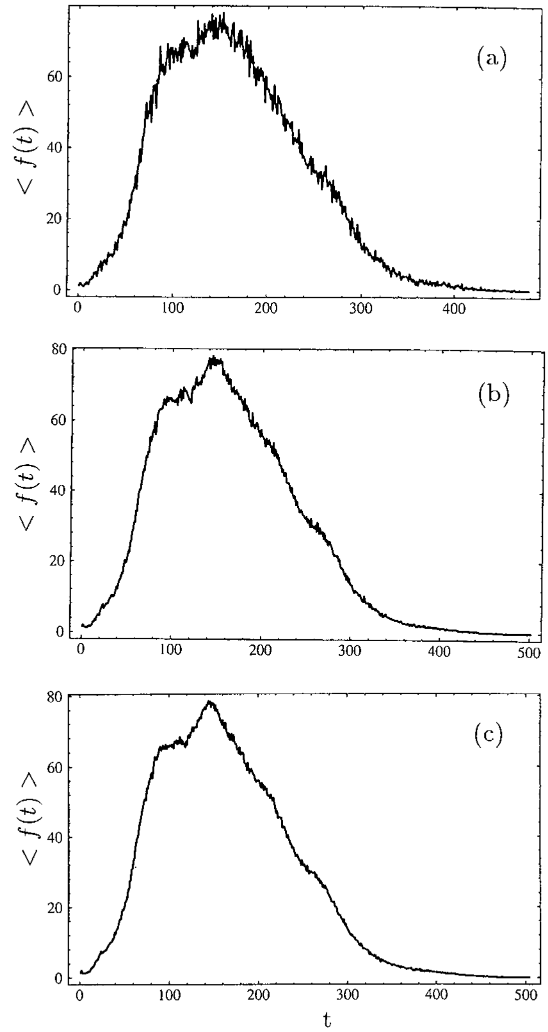

Figure 8. The effects of progressive MC averaging on $\langle f(t) \rangle$. Here $u_c/u_h = 8$, $u_c = 1$, and the variance of the (mean $\frac{1}{8}$) lognormal distribution for u_h is 0.5: (a) 10 MC realizations; (b) 50 MC realizations; (c) 100 MC realizations.

Figure 8, where $u_c/u_h = 8$, $u_c = 1$, and the variance of the (mean $\frac{1}{8}$) lognormal distribution for u_h is 0.5. A smoothing of the response is progressively achieved, while the main features remain unchanged, as they are most sensitive to the ratio u_c/u_h. Figure 9 shows the dramatic changes that mean, variance, and the dimensionless third moment of the response undergo as a function of the ratio u_c/u_h in the deterministic case.

Many other cases have been tested for river basins in northern Italy and comparisons with real runoff data have been performed. Since the results are not central to the issue at hand, we will report them elsewhere (Rodriguez-Iturbe et al., manuscript in preparation, 1995) aiming at rainfall-runoff analyses in the above framework. What matters here is that different dynamics can mix considerably the geometric and topologic information contained in the width functions. Since the dynamic characters can vary considerably in time and space (e.g., depending on initial moisture conditions, vegetation state, rainfall intensity, and spatial distribution), the fact that they have considerable impact on the response seems to limit considerably the capabilities of the inverse procedure, that is, from gauging to morphology.

Conclusions

The morphologies of river basins contain important information on the features of their hydrologic response. High-frequency modes of geomorphologic width functions have recursive characters notwithstanding the variety of geologic, climatic, vegetational, and geomorphic features in real river basins. These recursive characters reflect the common pattern of self-organization that river networks experience. These high-frequency modes do not bear fundamental implications in the determination of the hydrologic response, that is, being smoothed and tending to lose the original fractal characters via the filtering effect of the dynamics. The recurrence of such characters, well reproduced by optimal channel networks (OCNs), allows for the reconstruction of width functions from the outer shape of the basin. We have also shown in this paper that the shape of the basin can be effectively recovered from the width function of the embedded network.

On the contrary, the hydrologic response is imprinted in the low Fourier modes of the width function. These reflect the gross availability of contributing areas at isochrone distances from the outlet and are also accurately reproduced by OCNs.

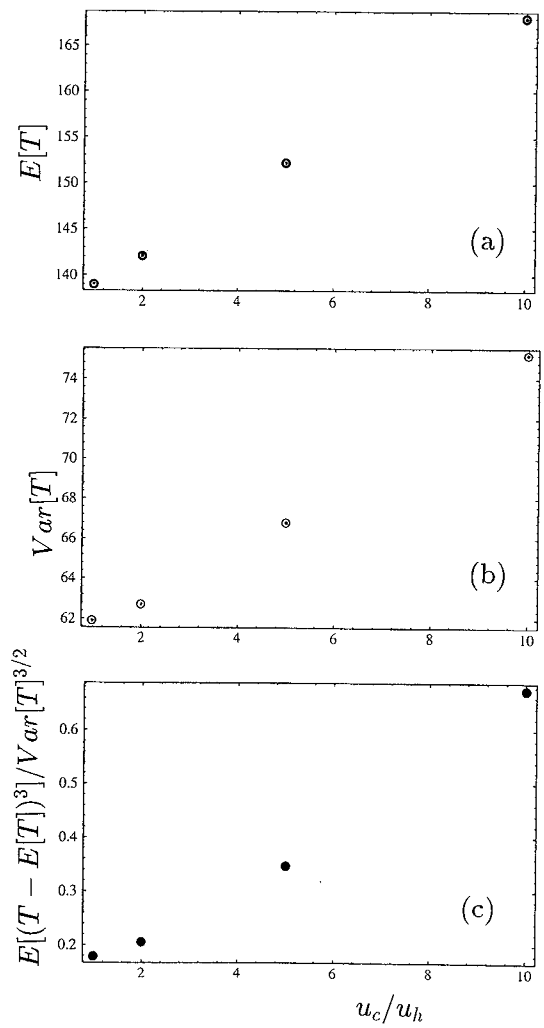

Figure 9. Evolution of mean $E[T]$, variance $\mathrm{Var}[T]$, and the dimensionless third moment of the travel times as a function of the dynamic ratio u_c/u_h for the Fella river basin.

However, isochrone distances are greatly affected by different dynamic specifications reflecting heterogeneity, hydrodynamic dispersion, and the distinction between hillslope and channel flow paths. These dynamic specifications make it impossible to recover the outer shape of the basin from the hydrologic response. Also, since a deterministic, constant drift is a rather crude representation of the dynamics in both river channels and hillslopes, it is believed that stochastic averaging explains the physical mechanism responsible for the smoothing which is typical of usual gauged records. The universal positive skewness of the response is suggested to be a by-product of differences in the mean drift in hillslopes and channels and of hydrodynamic dispersion.

It is concluded that the hydrologic response is imprinted in the shape of the basin. From known morphological features one can accurately predict the width function of the basin and through it devise many properties of the response by hypothesizing reasonable dynamic scenarios. However, the inverse procedure is less reliable, although we have shown that from a given width function one can obtain a robust estimation of the shape of the basin. The weak link is from gauged records to the width functions which are most affected by dynamic specifications irretrievable from the sole gauge trace.

Acknowledgments. This research was supported by CNR through Gruppo Nazionale per la Prevenzione Delle Catastrofi Idrometereologiche, Linea 3 (Progetti MIAP-METEO).

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(Received August 9, 1994; revised November 30, 1994; accepted December 12, 1994.)